is attributable to the tension in the beam as a result of its deformed shape. For all frequency values presented in Table 2, the corresponding mode shapes, measured relative to the deformed equilibrium state, were almost identical to the mode shapes obtained for the classical frequencies measured relative to the undeformed (straight state).

Conclusions

The results of this investigation indicate that significant changes in the vibration characteristics of beams can occur depending on their flexibility and prestressed state. The results also showed that frequency predictions of beams by classical theory can be in considerable error if the deformed equilibrium state is such that it can only be accurately described by nonlinear theory.

Finally, the effect of the weight component on the frequencies has been shown to be significant for the more flexible beams, indicating that appropriate consideration should be given to the design of such structural components if they are to be used in a weightless environment.

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Transition and Turbulence Phenomena in Supersonic Wakes of Wedges

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1. Introduction and Objectives

At supersonic speeds the wake of a blunt body may be divided into an inner viscous wake stemming from the body boundary layers and an outer "inviscid" wake produced by the bow shock. In such wakes, as in the wake of a cylinder,[1] at low Reynolds numbers the onset of transition occurs thousands of diameters downstream of the body in the outer, shock-induced wake. As the Reynolds number is increased, transition occurs in the inner wake. For the same Mach number and at high Reynolds numbers transition in the wake of a wedge[2] occurs further downstream than in the case of a cylinder, but the slender body transition curve crosses the blunt body transition curve.[3] The question arises, where does transition occur in slender body wakes as the Reynolds number is decreased further? What effect does the much weaker outer shock-induced wake have on transition and, is the outer wake unstable enough to become turbulent?

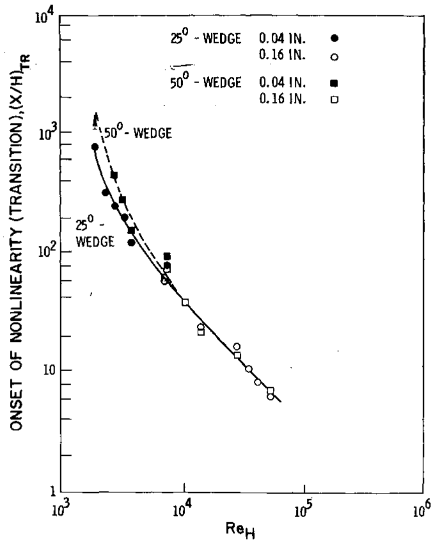

Fig. 1 Transition in wedge wakes at $M = 4.5$.

The first objective of the present work is to locate transition over a large range of Reynolds numbers in the wake of wedges and to investigate the effect of the outer, shock-induced wake on transition. Incompressible and hypersonic transitional wake flows may be divided into the linear instability region, the nonlinear instability region where a strong interaction between mean and fluctuating flow occurs, but a regular structure is still present, and the turbulent wake.[1,4,5] A study of the axial development of frequency spectra of fluctuations in these three regions is the second objective of this study.

The experiments were performed in the Jet Propulsion Laboratory's 20-in. supersonic wind tunnel at $M_\infty = 4.5$. Two wedges of 12.5° and 25° half angles, each of two sizes ($H = 0.04$ in. and 0.16 in.), were chosen for the study. The Reynolds number could be varied from $Re_H = 1900$ to 55,000. Hot wire measurements were made with a 0.0001-in. platinum-10% rhodium hot wire at constant current (for details see Ref. 3).

2. Transition Location

In the linear instability region fluctuations grow exponentially, and the mean flow still obeys the steady laminar boundary-layer equations. The wake centerline fluctuation signal is zero. Beyond a certain axial location the wake grows rapidly, indicating the onset of a strong interaction between the mean and fluctuating flow. This behavior is accompanied by the growth of fluctuations on the wake axis. This onset of nonlinearity will be called "transition." Transition location as a function of Reynolds number and wedge

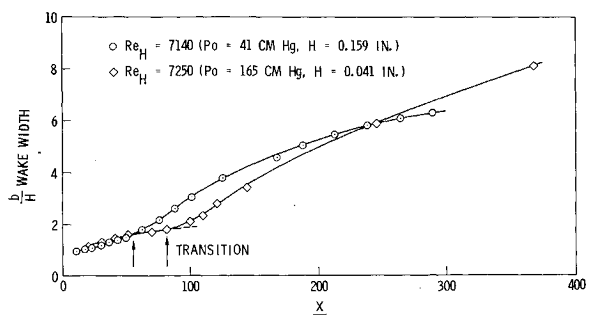

Fig. 2 Wake transition—effect of unit Reynolds number.

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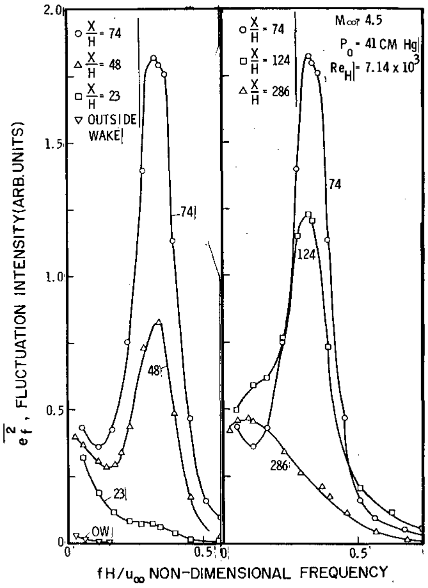

Fig. 3 Energy spectra of fluctuations in transitional wake of wedge.

angle is shown in Fig. 1. At large Reynolds numbers ($Re_H > 10^4$) the transition location does not appear to differ greatly for the two wedges. However, at a Reynolds number of $Re_H = 1900$, transition was found for the slender 12.5° wedge at $X/H = 780$ and was not found for the 25° wedge for $X/H \leq 1100$. Thus, the slender wedge has a markedly more unstable wake. An explanation of this effect was attempted on the basis of linear stability theory.[3]

These experiments did not indicate a substantial growth of flow fluctuations in the outer, shock-induced wake. Hence, the outer shock-induced wake of the wedges was not unstable enough to become turbulent. This result further suggests that at $M = 4.5$, $Re_H = 1900$ is near the critical Reynolds number where turbulence does not occur anymore in the wakes of "slender" wedges.

3. Unit Reynolds Number Effect on Transition

The wake widths at one Reynolds number ($Re_H = 7140$) for 12.5° wedges with a difference in base height of a factor of 4 are shown in Fig. 2. In the laminar region the normalized wake widths b/H are identical. However, transition occurs sooner (in terms of X/H) in the large wedge/low unit Reynolds number case than in the small wedge/high Reynolds number case.

This phenomenon is explained on the basis of linear stability theory and the knowledge of the level of fluctuations and frequency distributions in the freestream. For the present experiments both the level of fluctuations ($\bar{p}'/\bar{p}$), and the frequency spectrum stayed nearly the same for the whole pressure range.[6,7] However, the most unstable frequency is a function of the model size and for the large wedge $f \sim 60$ kHz compared to the small wedge where $f \sim 240$ kHz. A characteristic freestream spectrum shows a decrease in fluctuation energy with increasing frequency.[7] Therefore, the initial level of fluctuations in the most unstable frequency region, is larger for the large wedge wake. Since the growth rate of fluctuations is the same for both wakes, transition is expected to occur sooner (in terms of X/H) in the wake of the large wedge which is in agreement with observation (Fig. 2).

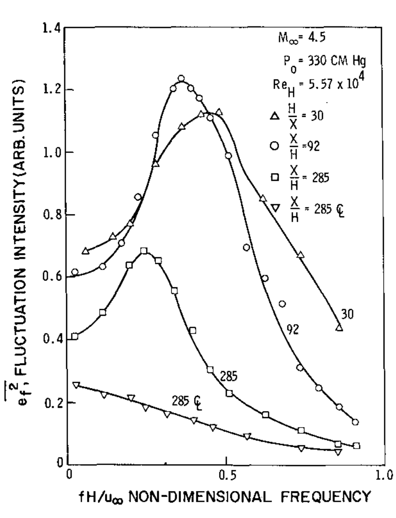

Fig. 4 Energy spectra of fluctuations in turbulent wake of wedge.

4. Frequency Distributions in the Transitional and Turbulent Wake

At $Re_H = 7140$, the frequency distributions of fluctuations were measured at the location of peak intensity. As shown in Fig. 3, out of a spectrum decreasing monotonically with increasing frequency near the wedge ($X/H = 23$) a large peak in the spectrum develops at $f \simeq 60$ kHz corresponding to a nondimensional frequency $fb_0/u_\infty \simeq 0.31$, where b_0 is the wake width near the body (at $X/H = 10$). This development of a pronounced peak in the spectrum is characteristic of linearly unstable wakes, and the value of the nondimensional frequency $fb_0/u_\infty \simeq 0.31$ is very near the value found for both incompressible and hypersonic wakes.[5] At the beginning of the nonlinear region ($X/H \simeq 55$) the fluctuation energy peak still increases, but then decreases, rapidly downstream and at $X/H = 286$ the peak in the spectrum has disappeared and a "turbulent" spectrum remains. The development of the spectrum is qualitatively similar to the development of the spectra measured by Roshko[8] in a vortex street behind a cylinder at low speeds. The frequency spectrum at $X/H = 286$ is quite similar to an empirical turbulent spectral distribution suggested by Dryden,[3] a result also found by Roshko in the low-speed wake.

At the much higher Reynolds number of 55,700 transition occurs near the body ($X/H \simeq 7$). In this turbulent wake, a pronounced peak in the spectrum develops at $fH/u_\infty \simeq 0.42$ ($fb/u_\infty = 0.55$) out of a spectrum with most of the energy at low frequencies (Fig. 4).

Previously a peak in the fluctuation spectrum in a turbulent wake has been observed[9] where $fH/u_\infty \simeq 1.27$ and $fb/u_\infty \simeq 1.0$. These results indicate that the preferred (peak) frequency of the turbulent wake does neither scale with the wake width nor with the wedge height and at present the phenomenon is not understood.

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